FAILURE UNDER ALTERNATING LOADS

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by

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TABLE OF SYMBOLS

b	Width of section at neutral axis
c	Distance from neutral axis to outermost fiber
d	Diameter
E	Modulus of elasticity
e_1 e_2 e_3	Principal strains
I	Moment of inertia
Ip	Polar moment of inertia of cross section
K	Theoretical stress concentration factor
k	Fatigue stress concentration factor
M	Bending moment
Mt	Torque
p	Uniform Tension
Q	Firt Moment of the area about neutral axis
q	Sensitivity index
r	Radii

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U	Strain energy per unit volume
v	Shear Force
Vo	Distortion energy per unit volume
α	Equivalent stress
θ	Phase angle
μ	Poisson's ratio
σ	Mean normal stress
σ_v	Variable normal stress
$\sigma_1, \sigma_2, \sigma_3$	Principal stresses
σ_m	Mean stress
σ_{max}	Maximum stress
σ_{min}	Minimum stress
σ_e	Endurance or fatigue limit
σ_{yp}	Yield point stress
σ_u	Ultimate tensile stress
$\sigma_1', \sigma_2', \sigma_3'$	Principal stresses, maximum value
$\sigma_1'', \sigma_2'', \sigma_3''$	Principal stresses, mean value

τ Mean shear stress

τ_v Variable shear stress

τ_{oct} Octahedral shearing stress

τ_s Shearing stress

CHAPTER I

INTRODUCTION

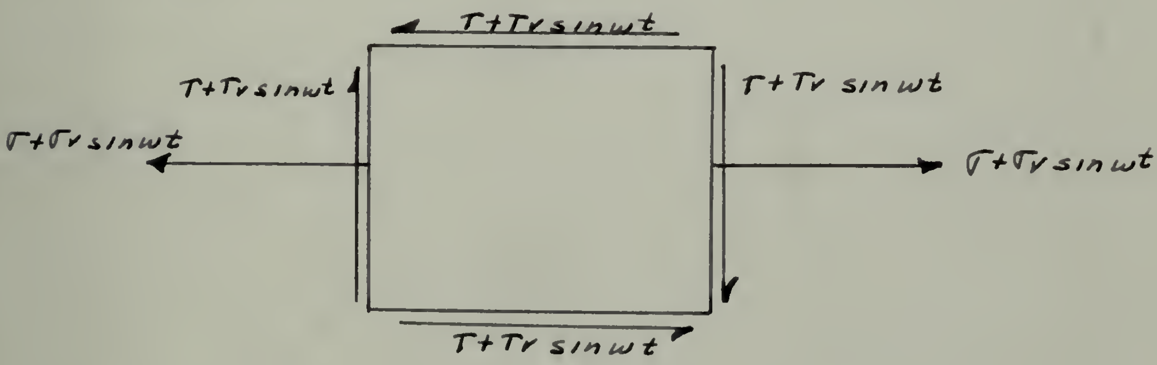
The object of this thesis is to acquire a working knowledge of the principles used in the design of rotating shafts subjected to combined fatigue stress.

The treatment is divided into two parts: first, an investigation of the problems solely from the standpoint of stress; second, a discussion of the effects of stress concentration, hardness of material, and surface treatment in the determination of working stress.

The treatment will be restricted in that:

1. Only axial and shear stresses will be considered, as shown for an element in the diagram below.

2. Materials will be regarded as ductile, isotropic, and homogeneous.



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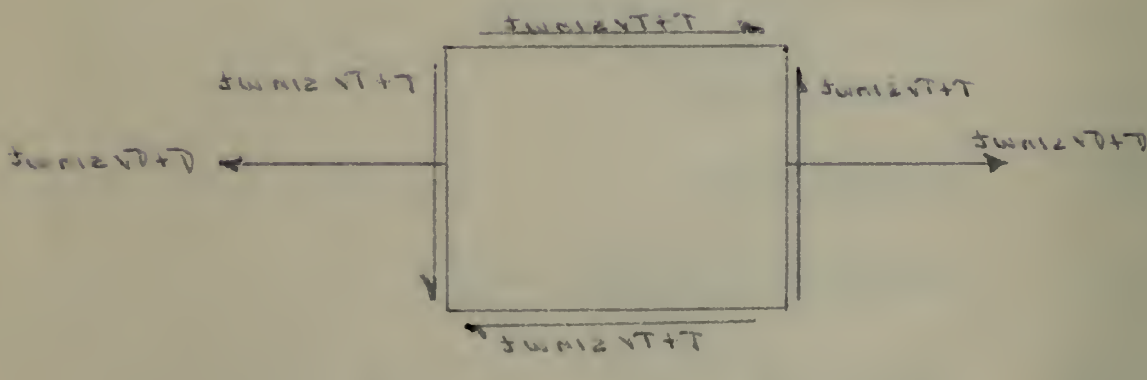

In arriving at the general relations for the allowable working stress in shafting, two sub-cases will be discussed:

(a) Axial stress varying between maximum and minimum values while the shear stress remains constant.

(b) Axial stress and shear stress varying between maximum and minimum values of different magnitudes and in phase.

Note that the axial and shear stresses can be computed using the standard formulas: $\frac{Mc}{I}$, $\frac{P}{A}$; $\frac{Mcr}{I_p}$; $\frac{VQ}{Ib}$.

CHAPTER II

THEORIES OF FAILURE

For the case of static stresses, the allowable stress can be determined in terms of the principal stresses using one of the theories of failure. In discussing the various theories, only those which are in near agreement with actual tests will be presented as they not only predict with more accuracy failure from static combined stresses, but also failure due to fluctuating stresses. Failure is defined as the beginning of inelastic action (yielding).

The treatment of the theories of failure will consider the most general case, that is, the stress condition of an element of a body is defined by the magnitudes of the principal stresses σ_1, σ_2 and σ_3. For convenience, we presume $\sigma_1 \geq \sigma_2 \geq \sigma_3$.

The Maximum Shear Theory, as first suggested by Coulomb, assumes that failure of materials subjected to combined stresses is due to shear rather than direct stress. To lend support to this assumption, the physical appearance of material after being subjected to load reveals the presence of so-called slip layers of fine markings on the surface of the deformed bodies which approximately coincide with the planes of maximum shearing stress. As Nádai (1) points out, the fine line markings were interpreted as the intersections of thin layers of material with the surface of the deformed pieces, in which the grain structure of the substance was distorted through the yielding. These planes in certain materials are inclined at an angle of 45 degrees with respect to the directions

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of the largest and smallest principal stress. Based on this assumption, Guest later formulated the maximum shear theory. By this theory, failure occurs when the maximum shear stress in an element subjected to combined stresses reaches the value of the maximum shear stress at failure in simple tension.

It may be shown that the extreme shearing stress occurs on planes bisecting the dihedral angles between the principal planes. The magnitudes are

$$\frac{\sigma_1 - \sigma_2}{2}, \quad \frac{\sigma_2 - \sigma_3}{2}, \quad \text{and} \quad \frac{\sigma_1 - \sigma_3}{2}$$

Because of the convention adopted above, the greatest shearing stress is $\tau = \dfrac{\sigma_1 - \sigma_3}{2}$

Since from the limiting case of simple tension or compression, the maximum shear stress becomes, $\tau = \sigma/2$

The maximum shear theory predicts failure will occur when $(\sigma_1 - \sigma_3)/2$ becomes equal to the shear at failure in simple tension, or

$$\sigma_1 - \sigma_3 = \sigma$$

The <u>Maximum Strain Energy Theory</u>, as suggested by Beltrami, later formulated by Huber, and still later again by Haigh, predicts that failure is based on the concept of energy of deformation. It assumes that failure results when the total strain energy of deformation per unit volume, in the case of combined stresses, is equal to the strain energy per unit volume in simple tension. For gradually applied loads, the strain energy per unit volume is

$$U = \frac{\sigma_1 e_1}{2} + \frac{\sigma_2 e_2}{2} + \frac{\sigma_3 e_3}{2}$$

where e_1, e_2, and e_3 are the principal strains. On substituting the values for strains in terms of principal stresses, the strain

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$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

where E is the modulus of elasticity and μ is Poisson's ratio. Since at the beginning of elastic failure in simple tension, the unit strain is σ/E, the strain energy for simple tension becomes $\sigma^2/2E$. The theory then predicts that failure will occur when

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] = \frac{\sigma^2}{2E}$$

The <u>Maximum Distortion Energy Theory</u> was developed by Von Mises and Hencky. It assumes that failure begins in the case of combined stress when the energy of distortion or shear approaches the same energy at failure as in the case of simple tension. In the development of this theory, it is considered that the total strain energy (U) is divided into two parts: the energy to produce a change in volume and the energy used to distort the element. Only the second part is used in the development of this theory. The theory was brought about by the fact that isotropic materials can endure large hydrostatic pressures without yielding. To develop the theory, we first divide the principal stresses in two parts

$$\sigma_1 = \sigma_1' + p, \quad \sigma_2 = \sigma_2' + p, \quad \sigma_3 = \sigma_3' + p$$

where

$$p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

Since from this theory

$$\sigma_1' + \sigma_2' + \sigma_3' = 0$$

the stress condition $\sigma_1', \sigma_2', \sigma_3'$, produces only distortion and the change in volume depends entirely on the magnitude of the uniform tension (p), the part of the total energy due to a change in volume is

$$\frac{ep}{2} = \frac{3(1-2\mu)p^2}{2E} = \frac{1-2\mu}{6E}(\sigma_1 + \sigma_2 + \sigma_3)^2$$

where $e = e_1 + e_2 + e_3$. Subtracting the total energy due to a change in volume from the total strain energy as determined and using the identity

$$\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 = -\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]+\sigma_1^2+\sigma_2^2+\sigma_3^2$$

the part of the total strain energy due to distortion can be presented in the form

$$V = U - \frac{1-2\mu}{E}(\sigma_1+\sigma_2+\sigma_3)^2 = \frac{1+\mu}{6E}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right]$$

The distortion energy at failure in simple tension is obtained by placing $\sigma_2 = \sigma_3 = 0$ and $\sigma_1 = \sigma$ or

$$V = \frac{(1+\mu)}{3E}\sigma^2$$

For combined stress, we then have

$$V = \frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right] = \sigma^2$$

It is of interest to note that the expression within the brackets namely,

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2$$

is proportional to the square of the shearing stress on an octahedral plane (planes whose normals have direction cosines $\pm 1/\sqrt{3}$ with respect to the principal directions.)

Since the expression for the octahedral shearing stress is

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}$$

the condition for failure may be expressed as

$$\tau_{oct} = \sqrt{2}/3\ \sigma$$

CHAPTER III

SIMPLE FATIGUE STRESS

With the static criteria of failure established, a discussion of the manner in which an expression is determined for failure of materials subjected to simple axial fatigue stress is next presented, since both concepts are used in the development of theories of failure for materials subjected to combined fatigue stress.

For simple axial fatigue stress, the variation of stress with time is usually sinusoidal. It may be represented as in Figure 1 (see separate sheet), and expressed as:

$$\sigma = \sigma_m + \sigma_v \sin \omega t$$

The representation further shows that the values of the mean stress σ_m and the variable stress σ_v are

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \quad , \quad \sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

The stress is thus conveniently thought of as consisting of two parts: a reversed stress superimposed on a steady, or static, stress.

With this in mind, two limiting conditions of stress at failure are possible. For the first condition, $\sigma_v = 0$, the stress is entirely static and failure occurs when $\sigma_v = \sigma_{yp}$ (the yield point in simple tension or compression). For the second condition, $\sigma_m = 0$, failure results from complete reversal of stress repeated a large number of times. From this type of failure, the endurance limit or fatigue limit σ_e of a material is obtained.

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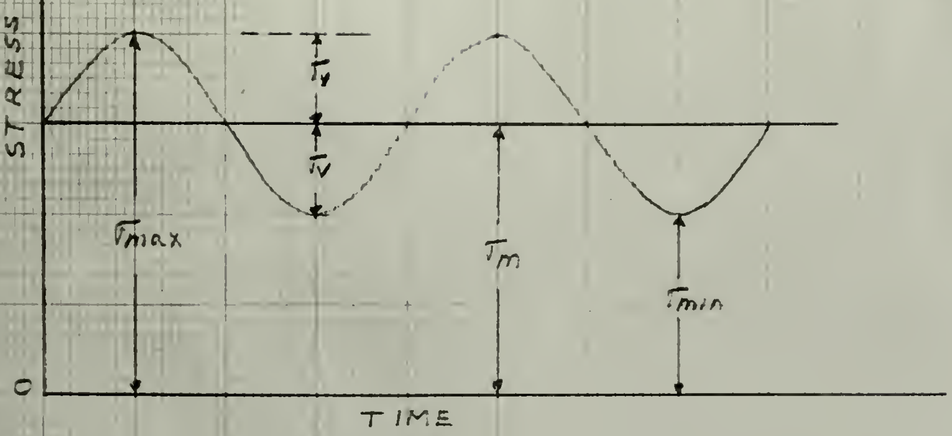
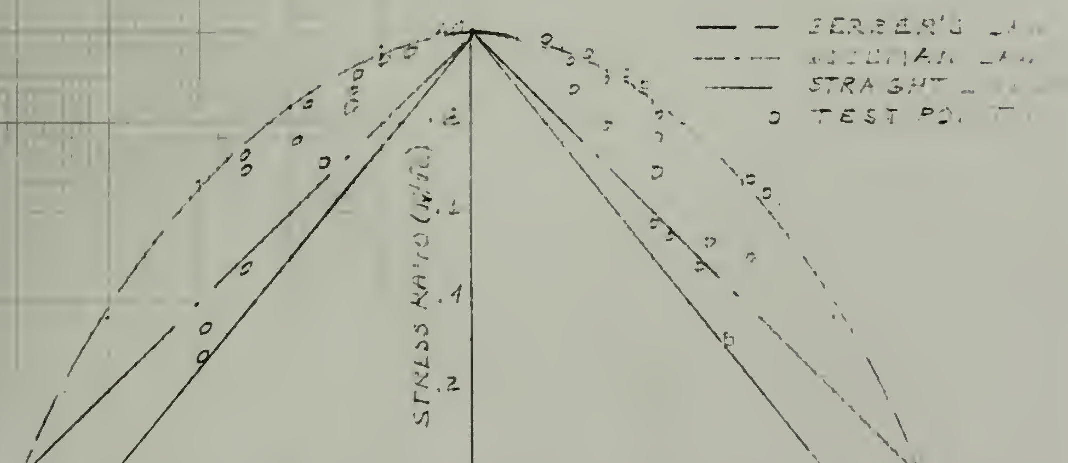


STRESS
0
TIME
τ_{max}
τ_m
τ_{min}
τ_v

FIG. 1



TEST
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
.2
.4
STRESS RATIO

FIG. 2

The endurance limit is considered as the maximum stress which a member can sustain for an indefinitely large number of cycles (usually taken as at least 10 million cycles for ferrous materials and about 50 million cycles for non-ferrous materials). Fatigue limits have been determined for various types of simple stresses such as alternating tension and compression, bending, and torsion for most of the materials in use today. It is of interest to note that the value of σ_e , as determined from tests of materials, Timoshewko (2), Moore and Koomers (3), is appreciably less than the value of σ_{yp} for the same material, thus the resisting strength of materials is reduced under the conditions of variable stress.

Since most problems present conditions intermediate between these extremes, it is necessary to consider all possible combinations of maximum and minimum stress or, more properly, it is necessary to consider the effect of mean stress on fatigue strength. A great deal of information can be obtained for variable axial stress and a typical diagram (Figure 2, see separate sheet) shows endurance limit ratios for a number of combinations of axial fluctuating stress, (σ_u is the ultimate tensile stress.)

Various attempts have been made to interpret such tests. The methods used reduce to empirical equations giving relations between the variable and mean stresses or the maximum stresses at failure. Of the various proposals, three are most used in design and are presented as follows:

<u>GERBER'S LAW</u> - Gerber's law is an empirical relationship which assumes that the relation for defining the variation of the variable stress with mean stress is of the parabolic form,

$$\frac{\sigma_v}{\sigma_e} = 1 - (\sigma_m/\sigma_u)^2$$

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or, in terms of the variable and mean stresses,

$$\sigma_v = \sigma_e - \sigma_e (\sigma_m/\sigma_u)^2$$

GOODMAN LAW - The Goodman Law is an empirical law which assumes that the relation defining failure for different combinations of variable stress and mean stresses is a straight line between the end points σ_v/σ_e and σ_m/σ_u. The equation for this straight line is

$$\sigma_v/\sigma_e = 1 - \sigma_m/\sigma_u$$

or, in terms of the variable and mean components,

$$\sigma_v = \sigma_e - (\sigma_m/\sigma_u)\sigma_e$$

STRAIGHT LINE LAW - The Straight Line Law proposed by Soderborg (4) assumes that this relation is defined by a straight line between the end points σ_v/σ_e and σ_{yp}/σ_m

The empirical relation is

$$\sigma_v/\sigma_e = 1 - \sigma_m/\sigma_{yp}$$

or, in a form which will be later used

$$\sigma_{max} = (1 - \sigma_e/\sigma_{yp})\sigma_m + \sigma_e$$

Of the three empirical relations, the Straight Line Law predicts with more accuracy failure under simple fatigue stress and will be used in the later developments of the combined fatigue stress theories. The major objection to the Gerber and Goodman Laws is that some of the test data falls below the empirical curves (on the unsafe side, indicated in Figure 2).

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CHAPTER IV

COMBINED FATIGUE STRESS

Under the conditions of variable stress, it was shown in the previous chapter that the resisting strength of a material is reduced. In the same way, the resisting strength in the case of combined stress will be modified. Since we are dealing with combined fatigue stress, the determination of working stresses for fluctuation loads will require the development of theories for defining failure as was done for the case of static combined stresses. The development of these theories will be presented in order to develop an accepted expression for the determination of working stress for the case under consideration.

The treatment of the theories of failure will consider the most general case as well as the case under consideration with one restriction, that is, only the case under consideration will be considered for the Shear Theory. For the general case, stress conditions on an elemental body are defined by the principal stresses $\sigma_1', \sigma_2', \sigma_3'$ and $\sigma_1'', \sigma_2'' \sigma_3''$ where the prime and double prime notation refer to the maximum and mean values of principal stress, respectively. The notation is similar to that used by Marin (5).

SHEAR THEORY - To develop a shear theory for failure, we first express the equation for defining failure

$$\sigma_{max} = (1 - \sigma_e/\sigma_{yp})\,\sigma_m + \sigma_e$$

in terms of fluctuating shearing stresses instead of fluctuation axial stresses.

The conversion can be accomplished by replacing σ_{max} and σ_m by $\tau_{max} = \sigma_{max}/2$ and $\tau_m = \sigma_m/2$ giving the equation

$$\tau_{max} = (1 - \sigma_e/\sigma_{yp})\,\tau_m + \sigma_e/2 \qquad (a)$$

Since for this theory only normal and shearing stresses will be considered, it is necessary in defining failure to consider the shear stress τ_s on any plane as shown for the element in question, Figure 3.

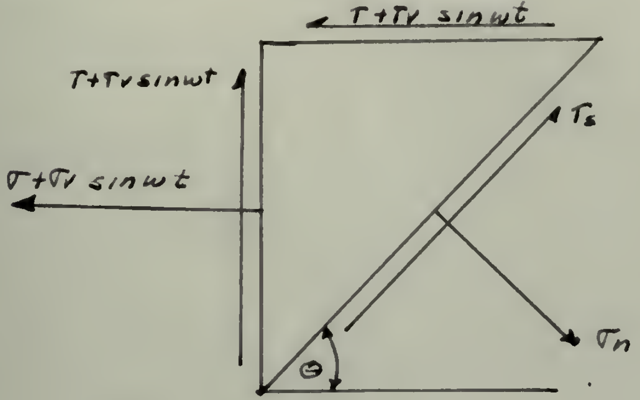

FIG. 3

For this particular plane, the maximum and mean components of shear stress are

$$\tau_{max} = \tfrac{1}{2}(\sigma + \sigma_v)\sin 2\theta + (\tau + \tau_v)\cos 2\theta$$

$$\tau_m = \tfrac{1}{2}\sigma \sin 2\theta + \tau \cos 2\theta$$

Failure for the plane, by the shear theory will occur when the above value of τ_{max} and τ_m satisfy Eq. a.

Therefore,

$$\left[\sigma + \sigma_v - \sigma(1 - \sigma_e/\sigma_{yp})\right]\sin 2\theta = 2\left[(1 - \sigma_e/\sigma_{yp})\tau - (\tau + \tau_v)\right]\cos 2\theta + \sigma_e$$

To determine the critical plane on which failure occurs, the above equation can be defined in terms of an equivalent stress α on any plane such that

$$\alpha = \left[(\sigma+\sigma_v) - \sigma(1-\sigma_e/\sigma_{yp})\right]\sin 2\theta - 2\left[(1-\sigma_e/\sigma_{yp})\tau - (\tau+\tau_v)\right]\cos 2\theta - \sigma_e$$

To find the most critical plane α should be a maximum. The maximum value of α ocurrs when

$$\tan 2\theta = \frac{(1-\sigma_e/\sigma_{yp})\sigma - (\sigma+\sigma_v)}{2\left[(\tau+\tau_v) - (1-\sigma_e/\sigma_{yp})\tau\right]}$$

On substituting the value of θ into Eq. (b) and combining terms, failure by the stress theory is expressed as

$$\sigma_e = \sqrt{\left[(\sigma_e/\sigma_{yp})\sigma + \sigma_v\right]^2 + 4\left[(\sigma_e/\sigma_{yp})\tau + \tau_v\right]^2}$$

<u>DISTORTION ENERGY THEORY</u> - The assumption to be made in the case of this theory is that failure occurs in the case of combined stresses when the distortion energy corresponding to the maximum value of stress components equals the distortion energy at failure for maximum axial stress. It is also necessary to require that this applies for equal values of distortion energy corresponding to the mean axial and mean combined stresses. To arrive at the above condition, it is necessary to define the fluctuating axial stress in terms of distortion energy, and use the relation as determined from the derivation of combined static stress. The distortion energy at any instant of time for simple axial stress as determined previously is

$$V = \left(\frac{1+\mu}{3E}\right)\sigma^2$$

From this equation, the values of distortion energy corresponding to the maximum and mean values of fluctuating stress are

$$V_{max} = \left(\frac{1+\mu}{3E}\right)(\sigma_{max})^2 \ ; \ V_m = \left(\frac{1+\mu}{3E}\right)\sigma_m^{\,2}$$

Since our equation for failure is

$$\sigma_{max} = (1 - \sigma_e/\sigma_{yp})\,\sigma_m + \sigma_e$$

failure in terms of distortion energies can now be expressed as

$$\sqrt{V_{max}} = (1 - \sigma_e/\sigma_{yp})\sqrt{V_m} + \left(\sqrt{\frac{1+\mu}{3E}}\right)\sigma_e$$

To get some general relations, the three dimensional case will again be considered since it was determined under theories of failure that distortion energy in terms of static principal stresses is

$$V = \left(\frac{1+\mu}{3E}\right)\left[\sigma_1^{\,2} + \sigma_2^{\,2} + \sigma_3^{\,2} - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)\right]$$

This generalization would give values for distortion energies corresponding to the maximum and mean components of stresses as

$$V_{max} = \left(\frac{1+\mu}{3E}\right)\left[(\sigma_1')^2 + (\sigma_2')^2 + (\sigma_3')^2 - (\sigma_1'\sigma_2' + \sigma_2'\sigma_3' + \sigma_1'\sigma_3')\right]$$

$$V_m = \left(\frac{1+\mu}{3E}\right)\left[(\sigma_1'')^2 + (\sigma_2'')^2 + (\sigma_3'')^2 - (\sigma_1''\sigma_2'' + \sigma_2''\sigma_3'' + \sigma_1''\sigma_3'')\right]$$

Failure by the distortion energy theory is now defined by substituting these expressions into the failure relation such that

$$\sigma_e = \sqrt{(\sigma_1')^2 + (\sigma_2')^2 + (\sigma_3')^2 - (\sigma_1'\sigma_2' + \sigma_2'\sigma_3' + \sigma_1'\sigma_3')} - (1 - \sigma_e/\sigma_{yp})\sqrt{(\sigma_1'')^2 + (\sigma_2'')^2 + (\sigma_3'')^2 - (\sigma_1''\sigma_2'' + \sigma_2''\sigma_3'' + \sigma_1''\sigma_3'')}$$

For the two dimensional case in which the combined stresses are the principal stresses, the distortion energy theory predicts (on letting $\sigma_3' = \sigma_3'' = 0$)

$$\sigma_e = \sqrt{(\sigma_1')^2 + (\sigma_2')^2 - (\sigma_1'\sigma_2')} - (1 - \sigma_e/\sigma_{yp}) \sqrt{(\sigma_1'')^2 + (\sigma_2'')^2 - \sigma_1''\sigma_2''}$$

To express this theory for the two dimensional stress components of Figure 3, it is only necessary to express the principal stresses in the above equations in terms of stress components. The substitution gives

$$\sigma_e = \sqrt{(\sigma + \sigma_v)^2 + 3(\tau + \tau_v)^2} + (1 - \sigma_e/\sigma_{yp}) \sqrt{\sigma^2 + 3(\tau)^2}$$

STRAIN ENERGY THEORY - Using the assumption that failure occurs as a result of total strain energy rather than distortion energy, another theory of failure can be developed. Since the manner of developing this theory is similar to that used for the distortion energy theory, the three dimensional case for strain energy may be expressed as:

$$\sigma_e = \sqrt{(\sigma_1')^2 + (\sigma_2')^2 + (\sigma_3')^2 - 2\mu(\sigma_1'\sigma_2' + \sigma_2'\sigma_3' + \sigma_1'\sigma_3')}$$

$$- (1 - \sigma_e/\sigma_{yp}) \sqrt{(\sigma_1'')^2 + (\sigma_2'')^2 + (\sigma_3'')^2 - 2\mu(\sigma_1''\sigma_2'' + \sigma_2''\sigma_3'' + \sigma_1''\sigma_3'')}$$

or in the two dimensional case where $\sigma_3' = \sigma_3'' = 0$.

$$\sigma_e = \sqrt{(\sigma_1')^2 + (\sigma_2')^2 + 2\mu(\sigma_1'\sigma_2')} - (1 - \sigma_e/\sigma_{yp}) \sqrt{(\sigma_1'')^2 + (\sigma_2'')^2 - 2\mu\sigma_1''\sigma_2''}$$

This equation may be used in applications where the normal stresses rather than the principal stresses are known. For such cases, it is only necessary to substitute for the values of principal stresses in terms of components in Figure 3 to conform with the stated problem. Making this substitution, the equation becomes:

$$\sigma_e = \sqrt{(\sigma+\sigma_v)^2+2(1+\mu)(\tau+\tau_v)^2} - (1-\sigma_e/\sigma_{yp})\sqrt{\sigma^2+2(1+\mu)\tau^2}$$

To summarize the above results, the equations for failure as depicted for the first two cases will be given in order to better compare them with test data,

Case (a) - Axial stress varying between maximum and minimum values while the shear stress remains constant. Failure will be predicted according to the theories as follows:

Shear Theory $$\sigma_e = \sqrt{(\sigma_e/\sigma_{yp}\sigma+\sigma_v)^2+4(\sigma_e/\sigma_{yp})(\tau^2)}$$

Distortion Energy Theory $$\sigma_e = \sqrt{(\sigma+\sigma_v)^2+3\tau^2} - (1-\sigma_e/\sigma_{yp})\sqrt{\sigma^2+3\tau^2}$$

Strain Energy Theory $$\sigma_e = \sqrt{(\sigma+\sigma_v)^2+2(1+\mu)\tau^2} - (1-\sigma_e/\sigma_{yp})\sqrt{\sigma^2+2(1+\mu)\tau^2}$$

Case (b) - Axial and shear stress varying between maximum and minimum values of different magnitudes and in phase. Failure will be predicted according to the theories as follows:

Shear Theory

$$\sigma_e = \sqrt{\left[(\sigma_e/\sigma_{yp})\,\sigma + \sigma_v\right]^2 + 4\,(\tau\,\sigma_e/\sigma_{yp} + \tau_v)^2}$$

Distortion Energy Theory

$$\sigma_e = \sqrt{(\sigma+\sigma_v)^2 + 3(\tau+\tau_v)^2} - (1-\sigma_e/\sigma_{yp})\sqrt{\sigma^2+3\tau^2}$$

Strain Energy Theory

$$\sigma_e = \sqrt{(\sigma+\sigma_v)^2 + 2(1+\mu)(\tau+\tau_v)^2} - (1-\sigma_e/\sigma_{yp})\sqrt{\sigma^2+2(1+\mu)\tau^2}$$

Since three different theories have been presented and appear justified for design, the next important question to be answered is: which of the three theories should be used in the design of shafting? In order to answer this question, it is necessary to rely on experimental test data, of which there is little for the problem under discussion.

To date, it appears that for ductile materials, the distortion energy theory more closely approaches test data than the maximum shear theory or strain energy theory. This is indicated by tests in fluctuating bending and torsion with complete stress reversals made by Gough and interpreted by Marin (6), Figure 4. In the plot of the test data σ_1 and σ_2 are the principal stresses and σ_e the endurance limit for simple tension and compression. The stress ratios σ_1/σ_e and σ_2/σ_e at which failure occurs under combined bending and torsion are indicated by small circles and dots. Experiments by Lea and Budgen, Figure 5 and Figure 6, were made on circular specimens subjected to static torque and completely reversed bending stress. Here again, it is of interest to note that the distortion energy theory more closely approaches test data than do the other theories.

Shear Theory

$$\sigma_e = \sqrt{\left[(\sigma_e/\sigma_p)\sigma + \sigma_v\right]^2 + 4\left(\tau\,\sigma_e/\sigma_p + \tau_v\right)^2}$$

Distortion Energy Theory

$$\sigma_e = \sqrt{(\sigma+\sigma_v)^2 + 3(\tau+\tau_v)^2 - (1-\sigma_e/\sigma_p)(\sigma^2+3\tau^2)}$$

Strain Energy Theory

$$\sigma_e = \sqrt{(\sigma+\sigma_v)^2 + 2(1+\mu)(\tau+\tau_v)^2 - (1-\sigma_e/\sigma_p)(\sigma^2+2(1+\mu)\tau^2)}$$

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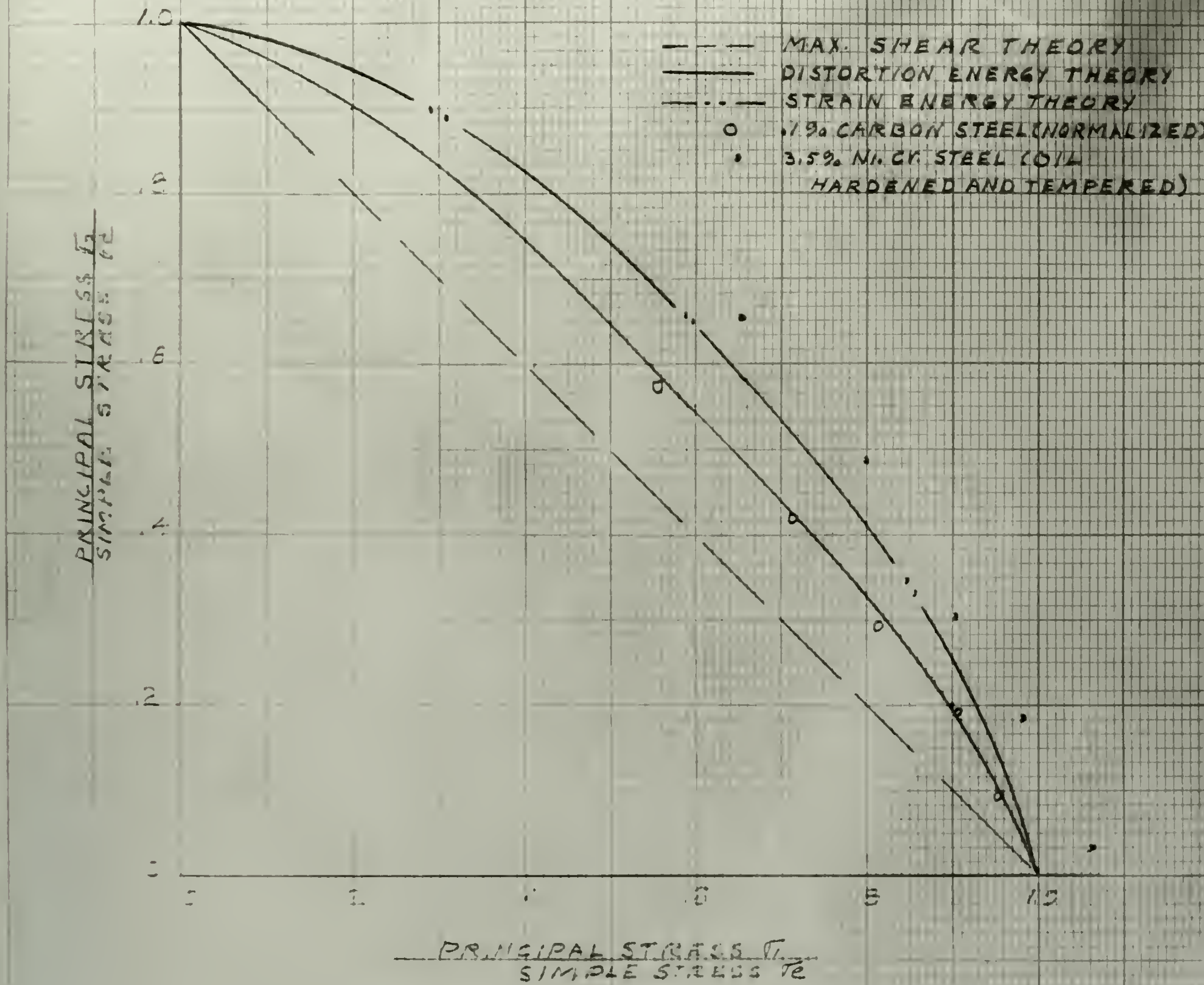
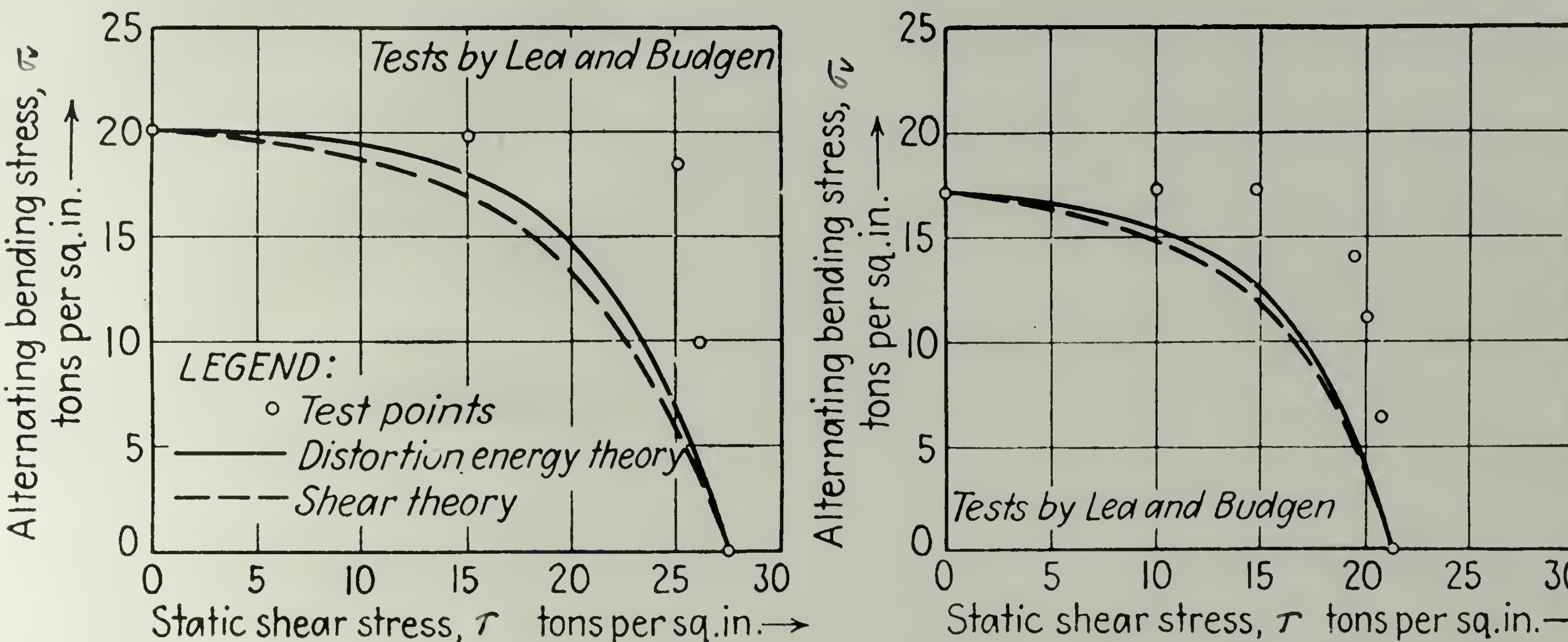


MAX. SHEAR THEORY
DISTORTION ENERGY THEORY
STRAIN ENERGY THEORY
o .19% CARBON STEEL (NORMALIZED)
• 3.5% Ni. Cr. STEEL (OIL HARDENED AND TEMPERED)
1.0
.8
.6
.4
.2
0
PRINCIPAL STRESS σ_2 / SIMPLE STRESS σ_e
PRINCIPAL STRESS σ_1 / SIMPLE STRESS σ_e
1.0

FIG. 4



(a)
Tests on Ni-Cr steel
(0.35%C, 0.6%Cr, 3.25%Ni)

(b)
Tests on "mild" steel
(0.14% carbon)

FIG. 5-6 -Combined fatigue-stress results for ductile metals.

Of interest at the present time are experiments conducted by Gough (7) which more closely approximate conditions set forth in the problem under discussion. His work consisted of determining an empirical equation for variable bending and torsion of different magnitudes in phase. The empirical expression (in terms of the problem under discussion) Gough presented for failure is:

$$(\tau_v/\tau_e)^2 + (\sigma_v/\sigma_e)^2(\sigma_e/\tau_e - 1) + \sigma_v/\sigma_e(2 - \sigma_e/\tau_e) = 1$$

Where σ_e is the endurance limit in pure bending, τ_e is the endurance limit in shear. As this equation is only developed for one material, it is of little assistance in present design, but indicates that research is underway to endeavor to determine a more exact expression for failure under condition of combined fatigue stress.

Since some basis for calculation of allowable stress under the assumed conditions has been established by the development of theories, to avoid failure in actual design, the stress imposed on shafting by operating loads must be less than this stress. To determine the relationship between the allowable stress, or working stress, in place of the stress components at failure, the axial stress at failure σ_e is replaced by its working stress value σ_e/n where n is the factor of safety.

The problem of choosing an adequate factor of safety in design is influenced by many factors. Some of these factors are:

1. Deficiencies in the theories of failure;

2. Assumption that materials are perfectly elastic, homogeneous, isotropic, and adhere to Hooke's Law;

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The problem of choosing an adequate factor of safety in design is influenced by many factors. Some of these factors are:

1. Deficiencies in the theories of failure;
2. Assumption that materials are perfectly elastic, homogeneous, isotropic, and adhere to Hooke's Law;

3. Machinery and fabrication errors;

4. Time effect, that is, no allowance is made in the determination of working stress for the deterioration of material with time due to corrosion, creep, electrolytic action, etc.;

5. Loads are rarely known accurately.

In consideration of these factors, it can be realized that the factor of safety must be greater than one (1), that is, the design must be overbalanced in favor of strength. Factors of safety are generally based on judgment and experience of the designer. For shafting, for example, the factor of safety has been judged to be between 2.25 and 3.00.

Since the method of determining the working stress has been considered, the equations defining failure by the various theories can be obtained. For comparison with static states of stress, both sides of the equation will be multiplied by σ_{yp}/σ_e such that working stresses defined by the various theories are:

Shear Theory

$$\sigma_w = \frac{\sigma_{yp}}{n} = \sigma_{yp}/\sigma_e \left[\sqrt{(\sigma_e/\sigma_{yp}\,\sigma + \sigma_r)^2 + 4\,(\tau\,\sigma_e/\sigma_{yp} + \tau_r)^2} \right]$$

Distortion Energy Theory

$$\sigma_w = \frac{\sigma_{yp}}{n} = \sigma_{yp}/\sigma_e \left[\sqrt{(\sigma + \sigma_r)^2 + 3\,(\tau + \tau_r)^2} - (1 - \sigma_e/\sigma_{yp}) \sqrt{\sigma^2 + 3\,(\tau)^2} \right]$$

Strain Energy Theory

$$\sigma_w = \frac{\sigma_{yp}}{n} = \sigma_{yp}/\sigma_e \left[\sqrt{(\sigma + \sigma_r)^2 + 2\,(1+\mu)\,(\tau + \tau_r)^2} - (1 - \sigma_e/\sigma_{yp}) \sqrt{\sigma^2 + 2\,(1+\mu)\,(\tau)^2} \right]$$

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Since the method of determining the working stress has been considered, the equations defining failure by the various theories can be obtained. For comparison with static states of stress, both sides of the equation will be multiplied by σ_{yp}/f_c such that working stresses defined by the various theories are:

Shear Theory

$$\sigma_w = \frac{\sigma_{yp}}{n} = \sigma_{yp}/f_c \left[\left(\frac{f_c}{\sigma_{yp}}\sigma + \sigma_v \right)^2 + 4\left(\tau \frac{f_c}{\sigma_{yp}} + \tau_v \right)^2 \right]^{1/2}$$

Distortion Energy Theory

$$\sigma_w = \frac{\sigma_{yp}}{n} = \sigma_{yp}/f_c \left[(\sigma+\sigma_v)^2 + 3(\tau+\tau_v)^2 - \left(1 - f_c/\sigma_{yp}\right)\left(\sigma^2 + 3\tau^2\right) \right]$$

Strain Energy Theory

$$\sigma_w = \frac{\sigma_{yp}}{n} = \sigma_{yp}/f_c \left[(\sigma+\sigma_v)^2 + 2(1+\mu)(\tau+\tau_v)^2 - \left(1 - f_c/\sigma_{yp}\right)\left(\sigma^2 + 2(1+\mu)\tau^2\right) \right]$$

CHAPTER V

In the design procedure presented in the previous chapter, the assumption was made that the shaft was of constant cross section or gradually varying cross section and that the shaft did not contain any discontinuities. However, no mention was made of any particular characteristics of the material such as hardness or the effect of surface treatment. In presenting the effects of these items on shafting, the topics will be taken up separately dividing the chapter into three parts.

PART I

STRESS CONCENTRATION

The effect of stress concentrations on metals subjected to alternating stress is of importance to engineers designing shafting because stress concentrations are invariably present due to fillets, holes, keyways, etc.

In the mathematical analysis of stress concentration based on the theory of elasticity for static loads, the results are usually stated in terms of a theoretical stress concentration factor:

$$K = \frac{\text{maximum stress of the section}}{\text{nominal stress of the section}}$$

since it is a function only of the geometry of the member for a

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$$K = \frac{\text{maximum stress of the section}}{\text{nominal stress of the section}}$$

since it is a function only of the geometry of the member for a

specific loading condition.

If the loads acting are alternating, the stress concentration factor can no longer be defined as above because test results show that the full effect of the theoretical stress concentration factor is realized in only a limited number of cases. The decrease of strength brought about by discontinuities is stated in terms of a fatigue stress concentration factor.

$$k = \frac{\sigma_e}{\sigma_e'} = \frac{\text{ordinary endurance limit without stress concentration}}{\text{endurance limit with stress concentration effect}}$$

Peterson (8) in a discussion of stress concentration, suggests another way of presenting this factor which is the percentage decrease (d) of endurance strength due to stress concentration:

$$d = \frac{\sigma_e - \sigma_e'}{\sigma_e} \times 100$$

To find some basis for "K" being less than "k", Peterson (8) evalued the principal of "stress concentration index" or sensitivity index which he expressed as the ratio:

$$q = \frac{k-1}{K-1}$$

The value of "q" ranges from zero (when k = 1.0 for certain tests of cast iron) to unity (should the fatigue stress concentration factor "k" attain the theoretical value "K").

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$$k = \frac{\text{ordinary endurance limit without stress concentration}}{\text{endurance limit with stress concentration effect}} = \frac{f_e}{f_c}$$

Peterson (3) in a discussion of stress concentration, suggests another way of presenting this factor which is the percentage decrease (d) of endurance strength due to stress concentration:

$$d = \frac{f_e - f_c}{f_e} \times 100$$

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There are many factors that influence the magnitude of the stress concentration effect in the case of fatigue. It has been

found by tests that while theoretical stress concentration factors are independent of the material, as it conforms to Hooke's Law, test data shows that fatigue stress concentration factors are not. Likewise, it appears that the variation of "k" for similar test pieces of different materials cannot be correlated with any of the ordinary properties of materials such as ductility and hardness. Although for steels, Peterson (8) presented test data which indicates a possible relationship existing between fatigue stress concentration factors and ultimate strength. Another important effect is the size of the discontinuity. If the material and size of a specimen or shaft are kept constant and the size of the discontinuity is varied, the theoretical stress concentration factor descreases as the size decreases. Fatigue stress concentration factors show a similar tendency except for a marked decrease for very small discontinuities. Still another important factor in the determination is the effect of size. Peterson found for various types of steel that there is very little variation in endurance limit in geometrically similar specimens without discontinuities. However, for geometrically similar specimens having holes, fillets, or artificial cracks, it was determined that small specimens have higher endurance limits than larger ones. This indicates a lower stress concentration factor for small elements. For example, in the case of circular shafts, with a transverse hole, of .45% carbon steel with a ratio of hole diameter to shaft diameter of 0.0625, the stress concentration factor determined experimentally for reversed bending increased from 1.33 to 1.84 when the shaft diameter was increased from .5 to 3.0 inches.

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With the knowledge that fatigue stress concentration factor

can be determined from test data, the important question is how should these factors be applied in design? Since Nádai (1) determined that materials have sufficient elasticity to allow for localized yielding under static loads, it may be assumed that stress concentration effects only the fatigue stress. Tests by Peterson verify this assumption. Therefore, in determining the working stress, stress concentration should be applied to the alternating component of stress and not to the static component. Since in this paper working stress is determined from the values of maximum and mean combined stresses, the effect of stress concentration is to leave the mean stress unchanged and to increase the alternating component, thereby increasing the maximum stress.

For actual values to use in design, references such as Lipson, Noll, and Clock (9), or Roark (10) may be consulted.

PART II

HARDNESS

Since endurance limits, which are a measure of the allowable stress under fatigue conditions, for all types of materials are unknown, a search has been made by engineers to determine if some correlation can be made between this property of metals and other properties that can be measured with comparative ease.

Among the mechanical properties that can be used to give a good estimate, hardness is considered by many to be the most

valuable for steels. Considering non-ferrous metals, there is too much scatter of results to justify any correlation for estimating endurance limit.

The one factor that makes this possible is that for static loading tests, the relationship between hardness and tensile strength, reference (11), is represented by a band that includes test data on alloys as well as plain carbon steels. The band width for all practical purposes can be considered negligible. That is, hardness may be considered proportional to the tensile strength.

In the case of endurance tests, the relationship between endurance limit and hardness was further complicated by the fact that tests showed that endurance limit did not only depend on hardness, but is also a function of the quality of the surface. For example, Hankins, Becker, and Mills (12), indicated that the variation in endurance limit between specimens finished with fine and coarse emery paper was 3% for a steel of 118,000 psi tensile strength and 11% for a steel of 138,000 psi tensile strength; and Hager (13) found no difference in endurance limit between rough and finished machined specimens of soft steel, the endurance limit of both being 10% less than for polished specimens. From a group of such tests (considering the fact that tensile strength is approximately proportional to hardness), Lipson, Noll, and Clock (9) devised a set of curves, Figure 7, showing the variation of endurance limit with hardness for materials whose surface conditions are in four groups; ground, machined, hot rolled, and forged. The ground surface finish includes ground, honed, lapped, and superfinished, and the machined surface finish includes rough and

valuable for steels. Considering non-ferrous metals, there is too much scatter of results to justify any correlation for estimating endurance limit.

The one factor that seems quite positive is that for static loading tests, the relationship between hardness and tensile strength, reference (11), is represented by a band that includes test data on alloys as well as plain carbon steels. The band width for all practical purposes can be considered negligible. That is, hardness may be considered proportional to the tensile strength.

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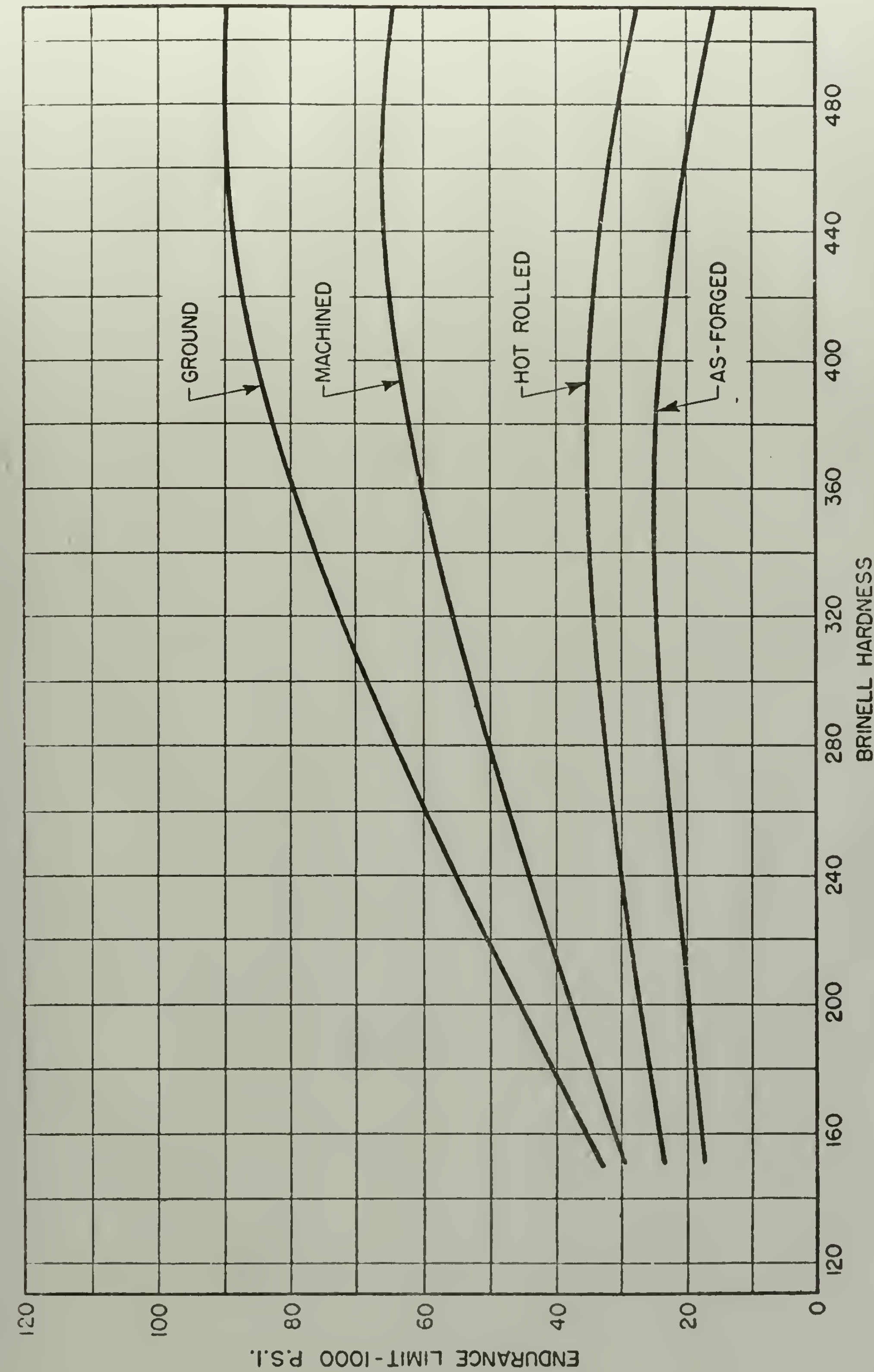

Fig. [illegible]. Relationship between hardness and strength for fatigue loading.

finished machined. Further evidence of the validity of these curves is given in tabular form by Eddy (9) for Brinell hardness ranging from 160 to 555.

PART III

SURFACE TREATMENT

Another factor which influences attainable stress is surface treatment. In most cases, the treating process is applied to metals to improve such properties as wear, corrosion resistance, etc.

The commonly used processes whose effects have been studied are cold working and surface hardening. These two processes will be considered separately.

COLD WORKING:

In cold working, the material is strained beyond the yield point and caused to flow plastically. The outstanding effects on metals are to raise the elastic limit markedly, noticeably increase the tensile stress, and decrease the ductility.

Cold working of metals appears to exert two opposing effects: (1) It elongates the crystalline grains in the direction of working into a more favorable position for resisting slip and fracture; (2) It tends to start new minute fractures, or tends to

set up severe internal stresses in the metal making fracture possible by a small additional applied stress. That is, for some degree of cold working, there is a maximum net benefit while for a more severe degree of cold working, the damage done increases more rapidly than does the benefit.

The most commonly used methods of cold working are cold rolling, stretching, and shot peening. The effect of all three of these processes appears to be an increase in the fatigue limit of materials. The amount of increase is usually a function of the condition of the surface before the process is applied. A variety of results has been obtained for the effect of cold stretching on the fatigue limit for non-ferrous metals. Tests (3) of brass and copper rod in which there is a reduction of area of 55% in a single pass of the cold drawing process showed no increase in fatigue limit over the limit of the same metal hot rolled. Tests of nickel and of other non-ferrous metals subjected to less drastic reduction, than that mentioned above, showed an appreciable increase in fatigue limit over the same metal annealed. For detailed data on non-ferrous metals, references such as Moore and Kommers (3) or Metals Handbook may be consulted.

For ferrous metals, the same general trend is noted. From a compilation of data (9), a general observation appears to be that, for any type of cold working, the minimum increase of fatigue limit is 2% for a polished hardened alloy after shot peening to 150% for a cold rolled machined specimen (SAE 1045 steel) after normalizing. Although insufficient data are available to make specific conclusions

on the beneficial effect of cold working on fatigue limit, the general trend can be noted in the case of shot peening. In the case of un-notched specimens, the increase in the fatigue limit for polished specimens seldom exceeds 20%, and in many cases, is less than 10%. For hot rolled specimens, the corresponding increase is 30 to 50% and for as-forged parts, 100%. The general trend is also apparent in the case of notched specimens, although the percentage improvement is higher.

SURFACE HARDENING:

Surface hardening is a process which increases the hardness of the surface to a depth ranging from a few thousandths of an inch to $\frac{1}{4}$ of an inch or more. Only a comparatively few number of tests have been conducted. They all show an increase in the endurance limit for ferrous and non-ferrous metals.

Remembering that endurance limit may be determined from hardness (Part II), Lipson, Noll, and Clock (9), presented a method for estimating endurance limits for surface treated ferrous parts. The method is based on the premises that, through hardness distribution over any cross section is non-uniform, the hardness distribution may be thought of as consisting of a hardened case and soft core. With this in mind to determine whether the case or core hardness should be used, the method proposed consists of superimposing the applied stress on the allowable stress as estimated by hardness. By using this method, it was determined that for estimating endurance limits, the hardness of the case should be used for un-notched machine parts while the hardness of the core

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SURFACE HARDENING:

Surface hardening is a process which increases the hardness of the surface to a depth ranging from a few thousandths of an inch to ¼ of an inch or more. Only a comparatively few number of tests have been conducted. They all show an increase in the endurance limit for ferrous and non-ferrous metals.

Remembering that endurance limit may be determined from hardness (Part II), Lipson, Noll, and Clock (9), presented a method for estimating endurance limits for surface treated ferrous parts. The method is based on the premise that, through hardness distribution over any cross section is non-uniform, the hardness distribution may be thought of as consisting of a hardened case and soft core. With this in mind to determine whether the case or core hardness should be used, the method proposed consists of superimposing the applied stress on the allowable stress as estimated by hardness. By using this method, it was determined that for estimating endurance limits, the hardness of the case should be used for un-notched machine parts while the hardness of the core

should be used for several notched machine parts. Test data substantiated this method for carbonized un-notched specimens (9). It can be surmized that it will apply to induction hardened and flame hardened machined parts. However, because of the nature of processing, there is between the case and core a transformation zone that is essentially in a normalized or annealed state. Its hardness may be less than the hardness of the core. Therefore, for conservative design, the hardness of the transition zone, rather than the hardness of core, should be used for un-notched or mildly notched machine parts. In the presence of sharp notches, the hardness of the case should be used for carbonized machine parts.

In the case of nitriding, tests have indicated that the endurance limit is unaffected by surface finish for un-notched or moderately notched specimens. Only very sharp discontinuities show any decrease in endurance limit. As a result, the above considerations are not applicable for nitriding; thus, endurance limits experimentally determined should be used in design.

For other discussions on the effect of surface treatment, see Peterson and Lessells (14), Woodvine (15), and Hoger (16).

CHAPTER VI

ILLUSTRATIVE PROBLEMS

In order to illustrate the use of the suggested concepts for the design of rotating circular shafts for cases (a) and (b), a group of problems is presented in which the diameter (d) is the desired quanity. The loading will be restricted to pure bending and torque. The distortion energy theory of failure will be used throughout since it was suggested in Chapter IV that it more accurately predicts failure when shafting is subjected to combined fatigue stress. The material is assumed to be a S.A.E. 1035 steel which has been quenched in water at 1525-1575°F with a Brinell hardness of 212. Since accurate data for yield point stress σ_{yp} and endurance limit σ_e is not known, values are taken from reference (9) ($\sigma_{yp} = 69000 psi$, $\sigma_e = 37000 psi$) To show the effect of stress concentration, a profile keyway is considered. The values used for fatigue stress concentration factors are those given in reference (9). In each problem, the safety factor (n) is considered to be 2.5. The rotating shaft is shown diagrammatically in Figure 7.

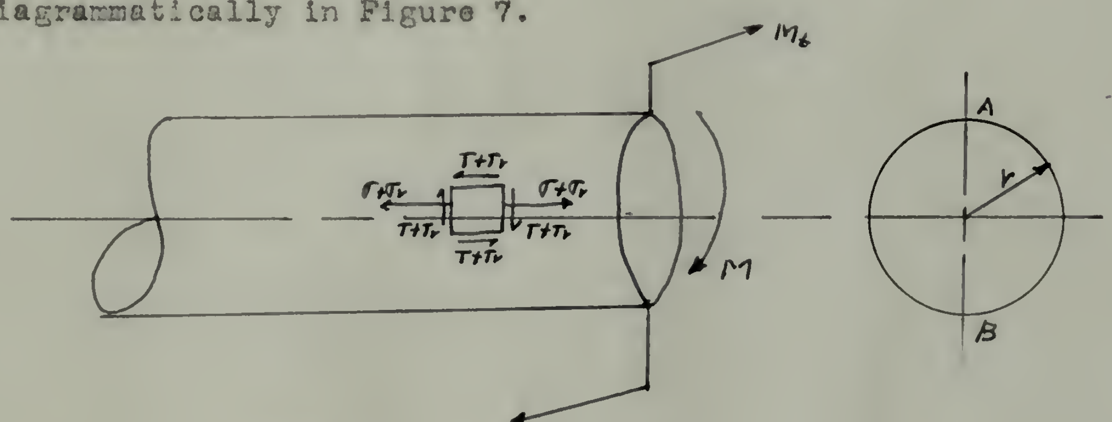

FIG. 7

CHAPTER VI

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In order to illustrate the use of the suggested concepts for the design of rotating circular shafts for cases (a) and (b), a group of problems is presented in which the diameter (d) is the desired quantity. The loading will be restricted to pure bending and torque. The distortion energy theory of failure will be used throughout since it was suggested in Chapter IV that it more accurately predicts failure when shafting is subjected to combined fatigue stress. The material is assumed to be a S.A.E. 1035 steel which has been quenched in water at 1525-1575°F with a Brinell hardness of 212. Since accurate data for yield point stress σ_{yp} and endurance limit σ_e is not known, values are taken from reference (4) ($\sigma_{yp} = 68000psi$, $\sigma_e = 37000psi$) To show the effect of stress concentration, a profile keyway is considered. The values used for fatigue stress concentration factors are those given in reference (9). In each problem, the safety factor (n) is considered to be 2.5. The rotating shaft is shown diagrammatically in Figure 7.

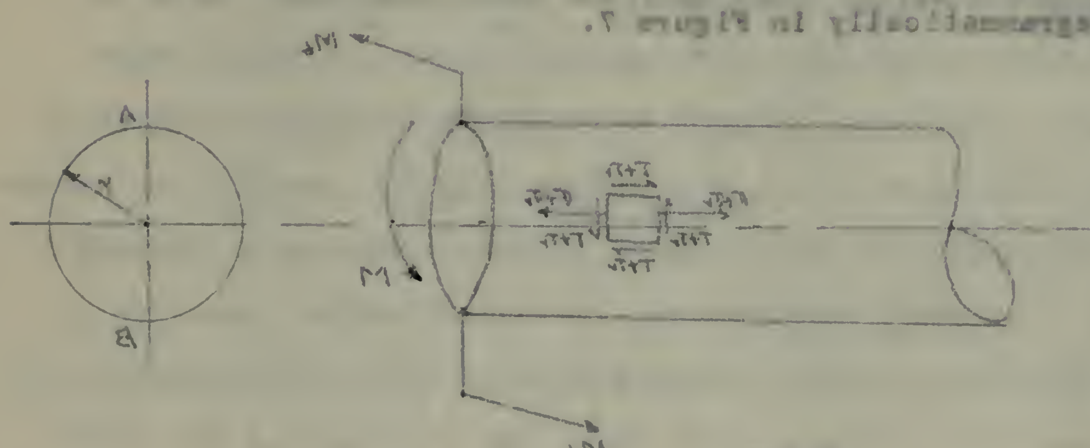

Fig. 7

ROTATING CIRCULAR SHAFT SUBJECTED TO FLUCTUATING PURE BENDING AND STATIC TORQUE

Let the value of bending moment vary from a maximum, $M' = 12000$ lb in, to a minimum, $M'' = 8000$ lb in, while the torque remains constant, $M_t = 6000$ lb in

The normal stress produced by the moment M will vary as the shaft rotates, and as M changes from a maximum, $M'r/I$ (point A, Figure 7), to a minimum, $-M'r/I$ (Point B, Figure 7). The values of maximum and mean normal stresses are:

$$\sigma + \sigma_v = \frac{32M'}{\pi d^3} = \frac{12.25 \times 10^4}{d^3} \text{ psi}, \quad \sigma = 0$$

The shear stress produced by the torque remains constant; thus, the maximum and mean values of stress are:

$$\tau + \tau_v = \tau = \frac{16M_t}{\pi d^3} = \frac{3.06 \times 10^4}{d^3}$$

Since the criteria of failure is:

$$\sigma_w = \frac{\sigma_e}{n} = \sqrt{(\sigma+\sigma_v)^2 + 3\tau^2} - (1 - \sigma_e/\sigma_{yp})\sqrt{3\tau^2}$$

substituting the values given above into this expression, the diameter is determined to be

$$d = 1.95 \text{ in}$$

ROTATING CIRCULAR SHAFT SUBJECTED TO FLUCTUATING PURE BENDING AND STATIC TORQUE WITH A PROFILE KEYWAY

Let the bending moment and torque remain the same as the previous problem. As a result, the only change in stress is that the maximum normal stress (same as the variable component as $\sigma = 0$) becomes $\sigma + K_b \sigma_v$. This follows the concept developed in Chapter V, that is, the fatigue stress concentration factor is applied only to the variable stress. The value of K_b (fatigue stress concentration for pure bending) from reference (9) is 2.0.

Since the criteria of failure is:

$$\sigma_w = \frac{\sigma_e}{n} = \sqrt{(\sigma + K_b\sigma_v)^2 + 3\tau^2} \;-\; (1 - \sigma_e/\sigma_{yp})\sqrt{3\tau^2}$$

substituting the values given into this expression, the diameter determined is d = 2.5 in.

Note due stress concentration the required diameter has increased by 30% as compared to the previous problem.

ROTATING CIRCULAR SHAFT SUBJECTING TO FLUCTUATING PURE BENDING AND FLUCTUATING TORQUE

Let the value of bending moment vary from a maximum $M' = 12000\,lb\,in$, to a minimum, $M'' = 8000\,lb\,in$, while the torque varies from a maximum, $M_t = 6000\,lb\,in$, to a minimum, $M_t'' = 4000\,lb\,in$

The normal stress produced by the varying moment will remain the same as for the first two problems, namely:

$$\sigma + \sigma_v = \frac{32M'}{\pi d^3} = \frac{12.25 \times 10^4}{d^3}\ lb\,in \quad , \sigma = 0$$

The shearing stress imposed by the torque will vary as the torque changes. The maximum mean and variable values of shear stress are:

$$\tau + \tau_v = \frac{16 M_t'}{\pi d^3} = \frac{3.06 \times 10^4}{d^3}, \quad \tau = \frac{16}{\pi d^3}\left(\frac{M_t' + M_t''}{2}\right) = \frac{2.55 \times 10^4}{d^3}$$

$$\tau_v = \frac{16}{\pi d^3}\left(\frac{M_t' - M_t''}{2}\right) = .51 \times 10^4$$

Since the criteria of failure is:

$$\sigma_w = \frac{\sigma_e}{n} = \sqrt{(\sigma + \sigma_v)^2 + 3(\tau + \tau_v)^2} - \left(1 - \sigma_e/\sigma_{yp}\right)\sqrt{3\tau^2}$$

substituting the values given into this expression, the diameter is determined to be d = 2.00.

ROTATING CIRCULAR SHAFT SUBJECTED TO FLUCTUATING PURE BENDING AND FLUCTUATING TORQUE WITH A PROFILE KEYWAY

The conditions of loading are considered to remain the same as in the previous problem. The only changes in stress are that the maximum normal stress (same as the variable stress as $\sigma = 0$) becomes $\sigma + K_b \sigma_v$ and the maximum shear stress becomes $\tau + K_s \tau_v$. The application of the fatigue stress concentration factor follows the concept developed in Chapter V, fatigue stress concentration factor is applied only to the variable stress. The values of K_b (fatigue stress concentration factor for pure bending) and K_s (fatigue stress concentration factor for applied torque) are, from reference (9), 2.0 and 1.6 respectively.

Since the criteria of failure is:

$$\sigma_w = \frac{\sigma_e}{n} = \sqrt{(\sigma + K_b \sigma_v)^2 + 3(\tau + K_s \tau_v)^2} - (1 - \sigma_e / \sigma_{yp}) \sqrt{3\tau^2}$$

substituting the values given into this expression, the diameter is determined to be d = 3.0.

Note due stress concentration the diameter has increased 50% as compared to the previous problem.

CHAPTER VII

CONCLUSIONS

Since to date, no exact solution is known for determining the failure of machine components subjected to fatigue stress, it is the author's opinion that the distortion energy theory gives the best approximation for varying tension, compression, and shear loads in phase.

From an analysis of the problem, the best analytical approach appears to be that stress concentration effects only the variable component of stress. Furthermore, the endurance limit seems to be a function of the hardness of, and surface treatment applied to, materials. Thus, it can usually be taken into the analytical approach depending on the previous history of the material.

Since the overall problem of fluctuating loads is of such great interest to designers, the writer feels that more research should be conducted in order to determine a more exact solution for the problem of combined fatigue stress.

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